Precision tests of the Standard Model and beyond

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Discovery of Neptune

Seen by Galileo, Lalande, Herschel, Lamont in 17/18th century but not identified as planet.

Around 1800, Lexell, Delambre, Bouvard and others noticed irregularities in orbit of Uranus → Caused by new planet?

1846: Le Verrier and Adams predict position, orbit and mass of Neptune

Observation by Galle in 1846
Perihelion shift of Mercury

Newtonian mechanics: Planets’ orbits are ellipses

Effect of other planets: Point of closest approach (perihelion) moves over time

Analysis of Mercury by Le Verrier in 1859: Disagreement between observation and Newtonian theory of ~ 40” per century

Explained by general relativity Einstein ’16

<table>
<thead>
<tr>
<th>Cause</th>
<th>Perihelion shift (arcsec/century)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pull of other planets</td>
<td>531.63 ± 0.69</td>
</tr>
<tr>
<td>General relativity</td>
<td>42.98 ± 0.04</td>
</tr>
<tr>
<td>Sum</td>
<td>576.61 ± 0.69</td>
</tr>
<tr>
<td>Observation</td>
<td>574.10 ± 0.65</td>
</tr>
</tbody>
</table>
Lamb shift

Quantum mechanics predicts $2s_{1/2}$ and $2p_{1/2}$ of $^1\!_1\!H$ to be degenerate, but measurement shows a small split

Lamb, Retherford '47

Explanation through quantum corrections

Bethe '47

→ Led to development of QED and modern field theory

Schwinger, Feynman '48,49
Precision tests of the Standard Model

Particle spectrum of the Standard Model

Mass

1 GeV

1 MeV

1 eV

0

3 gen. of fermions

bosons

Latest discoveries:
- bottom quark (1977, FNAL)
- $W/Z$ bosons (1983, CERN)
- top quark (1995, FNAL)
- $\tau$ neutrino (2000, FNAL)
- Higgs boson (2012, CERN)
### Particles and interactions

<table>
<thead>
<tr>
<th>Neutrinos</th>
<th>$\nu_e$</th>
<th>$\nu_\mu$</th>
<th>$\nu_\tau$</th>
<th>0</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Charged leptons</td>
<td>e</td>
<td>$\mu$</td>
<td>$\tau$</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$u$</td>
<td>c</td>
<td>t</td>
<td>$2/3$</td>
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<td>Quarks</td>
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<td>s</td>
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**Force carriers:**
- photon $\gamma$
- gluons $g_1, \ldots, g_8$
- weak bosons $W^+, W^-, Z^0$
- Higgs boson $H$, couples to all massive particles
Particles and interactions

Quantum fluctuations:
Virtual emission & re-absorption of all physical particles

\[
\begin{align*}
\text{electromagnetic (QED)} & \quad e^- & \rightarrow & \rightarrow e^- \\
\text{strong (QCD)} & \quad e^- & \rightarrow & \rightarrow \gamma \rightarrow e^- \\
\text{weak} & \quad e^- & \rightarrow W & \rightarrow \nu e \\
\end{align*}
\]
Particles and interactions

**Quantum fluctuations:**
Virtual emission & re-absorption of all physical particles
Particles and interactions

Quantum fluctuations:
Virtual emission & re-absorption of all physical particles

QCD: corrections of few 10%
em/weak: corrections of few %

→ Sensitivity to heavy particles
from precision measurements
without direct observation
$W$ mass

$W$-boson mass can be calculated from muon decay rate:

\[ \Gamma_{\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3} F\left(\frac{m_e^2}{m_\mu^2}\right) (1 + \Delta q) \]

QED corrections (2-loop)

Ritbergen, Stuart '98
Pak, Czarnecki '08

\[ \frac{G_F^2}{\sqrt{2}} = \frac{e^2}{8s_w^2 M_W^2} (1 + \Delta r) \]

electroweak corrections

\[ \mu^- \rightarrow \nu_\mu \nu_e e^- \]

Ritbergen, Stuart '98
Pak, Czarnecki '08
The muon decay in the Standard Model:

$$G_F^2 \sqrt{2} = \frac{e^2}{8s_w^2 M_W^2} \left(1 + \Delta r(M_Z, M_H, m_t, \ldots)\right)$$

Electroweak corrections (few %)

Can solve for

$$M_W = M_W(G_F, M_Z, M_H, m_t, \ldots)$$

Although $$m_\mu \ll m_t, M_H, \ldots$$, the muon decay rate is sensitive to $$m_t, M_H, \ldots$$ through quantum corrections.
**W mass**

**µ decay in Standard Model:**

\[
\begin{align*}
\mu^- & \rightarrow W^- + \nu_\mu + e^- \\
W^- & \rightarrow \nu_e + e^- \\
\mu^- & \rightarrow t + \nu_\mu + e^- \\
H & \rightarrow W^- + \nu_e + e^- 
\end{align*}
\]

\[
G_F^2 = \frac{\frac{G_F^2}{\sqrt{2}}}{8s_w^2M_W^2} (1 + \Delta r(M_Z, M_H, m_t, \ldots))
\]

electroweak corrections (few %)

Can solve for

\[
M_W = M_W(G_F, M_Z, M_H, m_t, \ldots)
\]

**Experiment:** Particle Data Group '18

\[
G_F = 1.1663787 \times 10^{-5} \text{ GeV}^{-2}
\]

\[
M_W = 80.376(33) \text{ GeV} \quad \text{(LEP)}
\]

\[
80.387(16) \text{ GeV} \quad \text{(TEV)}
\]

\[
80.370(19) \text{ GeV} \quad \text{(LHC)}
\]
$Z$ cross section and branching fractions

$e^+ e^- \rightarrow f \bar{f}$ for $E_{CM} \sim M_Z$:

- Mass $M_Z$
- Width $\Gamma_Z = \sum_f \Gamma_{ff}$
- Braching ratio $R_f = \Gamma_{ff}/\Gamma_Z$
- $\sigma^0 \approx \frac{12\pi \Gamma_{ee} \Gamma_{ff}}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} = \frac{12\pi}{M_Z^2} R_e R_f$

$\Gamma_{ff} = C \left[ (g_L^f)^2 + (g_R^f)^2 \right]$

$\sigma_{had} [\text{nb}]$

$E_{cm} [\text{GeV}]$

$M_Z$

$\Gamma_Z$

$\sigma^0$

$f = e, \mu, \tau, u, d, ...$
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\[
\Gamma_{ff} = C \left[ (g^f_L)^2 + (g^f_R)^2 \right]
\]
$Z$ cross section and branching fractions

$e^+ e^- \rightarrow f \bar{f}$ for $E_{\text{CM}} \sim M_Z$:

- Mass $M_Z$
- Width $\Gamma_Z = \sum_f \Gamma_{ff}$
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Comparison with experiment:

$\Gamma_{ll} = 83.984(86) \text{ MeV}$

$R_b = 0.2163(7)$

Particle Data Group '18
\textbf{Z-pole asymmetries}

Parity violation in $Z f \bar{f}$ couplings:
\[ g^f_L \neq g^f_R \]

\textbf{Left-right asymmetry:}
\[
A_{LR} \equiv \frac{1}{P_{e^-}} \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = A_e
\]
\[
A_f = \frac{2(1 - 4\sin^2 \theta^f_{\text{eff}})}{1 + (1 - 4\sin^2 \theta^f_{\text{eff}})^2}
\]
\[
\sin^2 \theta^f_{\text{eff}} = \frac{g^f_R}{2|Q_f|(g^f_R - g^f_L)}
\]
**Z-pole asymmetries**

**Forward-backward asymmetry:**

\[
A_{FB} \equiv \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3}{4} A_e A_f
\]

![Diagram of forward-backward asymmetry](image)

**Polarization asymmetry:**

Average \( \tau \) pol. in \( e^+e^- \to \tau^+\tau^- \)

\[
\langle P_\tau \rangle = -A_\tau
\]

![Graph of OPAL collaboration '01](image)
**Z-pole asymmetries**

**Forward-backward asymmetry:**

\[ A_{FB} \equiv \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3}{4} A_e A_f \]

\[ e^- \quad \overline{f} \quad \theta \quad f \quad e^+ \]

**Polarization asymmetry:**

Average \( \tau \) pol. in \( e^+e^- \rightarrow \tau^+\tau^- \)

\[ \langle \mathcal{P}_\tau \rangle = -A_\tau \]

**Comparison with experiment:**

\[ A_l = 0.1475(10) \]

Particle Data Group '12
Combination and SM fit

Constraints from fit of SM to *all* electroweak precision observables:

**LEP EWWG ’05**

- **Tevatron**
- **SM constraint 68% CL**
- **Direct search lower limit (95% CL)**

Graph showing the constraint on $M_t$ and $M_H$ over years from 1990 to 2005.

**Erler ’18**

Graph showing the constraint on $m_t$ and $M_H$ with various exclusions and direct measurements.

- $\sigma_had$, $R_l$, $R_q$ (1σ)
- $Z$ pole asymmetries (1σ)
- $M_W$ (1σ)
- Direct $m_t$ (1σ)
- Direct $M_H$
- All except direct $M_H$ (90%)

Table showing the fit of SM with various asymmetries and measurements.

- $A_l$(LEP)
- $A_l$(SLD)
- $A_{FB}^{0,b}$
- $M_W$
- $M_H$
- Fit w/o $M_H$
- LHC average

Results:

- $109^{+247}_{-66}$
- $40^{+34}_{-29}$
- $387^{+585}_{-169}$
- $60^{+56}_{-19}$
- $94^{+25}_{-22}$
- $125.7 \pm 0.4$
Radiative loop corrections

<table>
<thead>
<tr>
<th></th>
<th>$M_W$ [GeV]</th>
<th>$\sin \theta_{W,\text{eff}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>now</td>
<td>± 0.012</td>
<td>± 16</td>
</tr>
<tr>
<td>LHC</td>
<td>± 0.010</td>
<td>± 15</td>
</tr>
<tr>
<td>ILC/GigaZ</td>
<td>± 0.005</td>
<td>± 1.3</td>
</tr>
<tr>
<td>1-loop</td>
<td>± 0.450</td>
<td>± 1000</td>
</tr>
<tr>
<td>2-/3-loop QCD</td>
<td>± 0.070</td>
<td>± 45</td>
</tr>
<tr>
<td>ferm. 2-loop EW</td>
<td>± 0.050</td>
<td>± 90</td>
</tr>
<tr>
<td>bos. 2-loop EW</td>
<td>± 0.002</td>
<td>± 1</td>
</tr>
<tr>
<td>leading 3-loop</td>
<td>± 0.005</td>
<td>± 25</td>
</tr>
</tbody>
</table>

Experimental precision sensitive to 2-/3-loop effects

- Marciano, Sirlin '80
- Djouadi et al. '88
- Chetyrkin, Kühn, Steinhauser '95
- Freitas et al. '00, Awramik, Czakon '03
- Awramik, Czakon, Freitas, Weiglein '04
- Awramik, Czakon, Freitas '06
- Faisst, Kühn, Seidensticker, Veretin '03
**Theory calculations**

Experimental precision requires inclusion of **radiative corrections** in theory (1-loop, 2-loop, and partial 3-loop)

Integrals over momenta/energies in loop:

\[
\int d^4q_1 d^4q_2 \, f(q_1, q_2, p_1, p_2, \ldots, m_1, m_2, \ldots)
\]

Example:

\[
\int_{-\infty}^{\infty} d^4q \, \frac{\ldots}{[q^2 - m_t^2][(q + p)^2 - m_b^2]} = \lim_{\Lambda \to \infty} \frac{3g^2}{32\pi^2} m_t^2 \left( \log \frac{\Lambda^2}{m_t^2} + \frac{1}{2} \right) + O\left(\frac{p^2}{m_t^2}\right)
\]

- Individual integrals can be **divergent** ($\Lambda \to \infty$)
  - Only complete physical result finite
- Beyond 1-loop in general no analytical results

Berends, Böhm, Buza, Scharf '94
Numerical integration of loop integrals:

- **Divergent terms:** Remove e.g. through subtraction of simpler terms

\[ \int d^4 q_1 d^4 q_2 (f - f_{\text{sub}}) + \int d^4 q_1 d^4 q_2 f_{\text{sub}} \]

- **Stability and fast convergence:**
  - For \( n \)-dim. integral, perform some integrations analytically
  - Variables transformations to avoid singularities and peaks

References:

- Cvitanovic, Kinoshita ’74
- Awramik, Czakon, Freitas ’06
- Levine, Park, Roskies ’82
- Becker, Reuschle, Weinzierl ’10
- Bauberger ’97
- Freitas ’12
- Nagy, Soper ’03
- ...
Theory calculations: Status

Organization of calculation:

- Large number of diagrams and tensor integrals, $\mathcal{O}(100) - \mathcal{O}(10000)$
- Computer algebra tools for generation of diagram and algebra manipulations
- Efficient codes (C/Fortran) for numerical integration
- No multi-purpose program for ($\geq 2$)-loop calculations

Many seminal works on 1-loop and leading 2-loop corrections

- Veltman, Passarino, Sirlin, Marciano, Bardin, Hollik, Riemann, Degrassi, Kniehl, ...

Full 2-loop results for $M_W$, $Z$-pole observables

- Freitas, Hollik, Walter, Weiglein '00
- Awramik, Czakon '02
- Onishchenko, Veretin '02
- Awramik, Czakon, Freitas, Weiglein '04
- Awramik, Czakon, Freitas, Kniehl '08
- Dubovyk, Freitas, Gluza, Riemann, Usovitsch '16,18

Approximate 3- and 4-loop results (to $\rho$ parameter)

- Chetyrkin, Kühn, Steinhauser '95
- Faisst, Kühn, Seidensticker, Veretin '03
- Boughezal, Tausk, v. d. Bij '05
- Schröder, Steinhauser '05
- Chetyrkin et al. '06
- Boughezal, Czakon '06
Searching for new physics

- All constituents of SM have been discovered
- SM leaves important open questions:
  - dark matter
  - matter-antimatter asymmetry
  - gravity
  - ...
- No evidence of new particles at Large Hadron Collider (LHC)
- Electroweak precision tests can constrain (or reveal) physics beyond the SM
Constraints on New Physics

4th Generation

**SM** has 3 fermion generations

**Hypothetical extra generation:** $t', b', l_4, \nu_4$

Limits from electroweak precision observables:

**Eberhardt '13**

![Graphs showing limits from electroweak precision observables](image)
Constraints on New Physics

Two Higgs Doublet Model

**SM** has one Higgs doublet (4 components)

→ Higgs boson + 3 “eaten” by $W/Z$ bosons

**Second Higgs doublet:** Four new particles $H^0, A^0, H^+, H^-$

Limits from electroweak precision observables:

![Image]

$m_{H^+} = 500$ GeV

Eberhardt '13
Constraints on New Physics

(Infinitely) long list of models:

- Models with new interactions
- Models with more Higgs bosons
- Models with a composite Higgs boson
- Models with supersymmetry
- Models with extra dimensions
- Models with a dark sector
- etc.
Effective operator analysis

Model-independent analysis:
- Describe effect of new particle through **effective operators**
- Symmetries of SM permit only small number of possible operators

Wilson '69; Weinberg '79
Effective operator analysis

- New particles with large masses $m_X \sim m_Y \sim \Lambda$:
  Particle cannot be produced directly if $\Lambda > E_{\text{collision}}$

- Short-term (“virtual”) fluctuations due to uncertainty principle:
  \[ (\Delta E)(\Delta t) \geq \frac{\hbar}{2}, \quad \Delta E = \Lambda - E_{\text{collision}} \]
  Larger $\Lambda$ $\Rightarrow$ Smaller $\Delta t$ $\Rightarrow$ Smaller measureable correction

- Need higher precision to probe larger mass scales $\Lambda$
Effective operator analysis

Assuming generation universality:

- Electroweak precision tests put constraints on new physics at TeV scale → Complementary to LHC
- Proposed new $e^+e^-$ colliders (ILC/CEPC/FCC-ee) can improve reach by factor $\sim 10$

Pomaral, Riva '13
Ellis, Sanz, You '14
Future $e^+e^-$ colliders

- **International Linear Collider (ILC)**
  Int. lumi at $\sqrt{s} \sim M_Z$: 100 $\times$ LEP

- **Circular Electron-Positron Collider (CEPC)**
  Int. lumi at $\sqrt{s} \sim M_Z$: 1,000-10,000 $\times$ LEP

- **Future Circular Collider (FCC-ee)**
  Int. lumi at $\sqrt{s} \sim M_Z$: $> 10^5$ $\times$ LEP
An intriguing mystery

Muon anomalous magnetic moment

Spin-1/2 particle with charge $q$ has magnetic moment $\frac{q}{2m}g$

Dirac equation: $g = 2$

Quantum corrections: $a_\mu \equiv \frac{g_\mu - 2}{2} \neq 0$

Measured at BNL $g$–2 experiment:

$$a_\mu = (11659208.0 \pm 6.3) \times 10^{-10}$$
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$$a_\mu = (11 659 208.0 \pm 6.3) \times 10^{-10}$$

FNAL g–2 experiment target:

$$a_\mu = (\pm 1.6) \times 10^{-10}$$
An intriguing mystery

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## Muon $g-2$: SM theory prediction

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<th>QED $\mathcal{O}(\alpha^5)$</th>
<th>$a_\mu , [10^{-10}]$</th>
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<tr>
<td>11 658 471.88 ± 0.01</td>
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Aoyama, Hayakawa, Kinoshita, Nio '12
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![Diagrams](attachment:diagrams.png)
# Muon $g-2$: SM theory prediction

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A) $\gamma \rightarrow \mu h$

B) $\gamma \rightarrow e h$

C) $\gamma \rightarrow h h$

D) $h \rightarrow \mu$

The figure on the right shows the Feynman diagrams for the processes involving the muon and hadrons, with $\gamma$, $\mu$, and $h$ representing the photon, muon, and hadron respectively.
## Muon $g-2$: SM theory prediction

<table>
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<tr>
<th>Contribution</th>
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<th>Authors and LoC</th>
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<td>light-by-light</td>
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<td>Keshavarzi, Nomura, Teubner ’18</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Prades, de Rafael, Vainshtein ’09</td>
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<td>Nyffeler ’09</td>
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<tr>
<td></td>
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<td>Erler, Toledo Sanchez ’06</td>
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![Diagram showing Feynman diagram for muon $g-2$ calculation](image)

The Feynman diagram illustrates the process of muon decay, with the contributions from various terms and their permutations.
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</tr>
<tr>
<td>NLO</td>
<td>−9.87 ± 0.09</td>
<td>Jegerlehner ’17</td>
</tr>
<tr>
<td>NNLO</td>
<td>1.24 ± 0.01</td>
<td>Keshavarzi, Nomura, Teubner ’18</td>
</tr>
<tr>
<td>light-by-light</td>
<td>10.5 ± 2.6</td>
<td>Prades, de Rafael, Vainshtein ’09</td>
</tr>
<tr>
<td>Total</td>
<td>11 659 182.3 ± 4.3</td>
<td>Nyffeler ’09</td>
</tr>
<tr>
<td>Exp</td>
<td>11 659 208.0 ± 6.3</td>
<td>Erler, Toledo Sanchez ’06</td>
</tr>
</tbody>
</table>

→ $\gtrsim$ 3.5 standard deviations!
Muon $g-2$: Uncertainties

Difficulties with hadronic contributions:

- Quarks form bound states (hadrons), difficult to compute from first principles†

†Ongoing work using Lattice QCD:

Blum et al. ’15,18  Lehner et al. ’17
Chakraborty et al. ’16  Borsanyi et al. ’17
Della Morte et al. ’17
Muon $g-2$: Uncertainties

Difficulties with hadronic contributions:

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Muon $g-2$: Uncertainties

Difficulties with hadronic contributions:

- Quarks form bound states (hadrons), difficult to compute from first principles\(^\dagger\)

Hadronic contributions from $e^+e^-$ data

- \(^\dagger\)Ongoing work using Lattice QCD:
  - Blum et al. ’15, ‘18
  - Lehner et al. ’17
  - Chakraborty et al. ’16
  - Borsanyi et al. ’17
  - Della Morte et al. ’17

\(^\dagger\)Ongoing work using Lattice QCD:
- Davier et al. ’17
- Jegerlehner ’17
- Keshavarzi, Nomura, Teubner ’18
New physics: Simplified models

Introduce one or two new fields (spin $0, \frac{1}{2}, 1$; SU(2) singlet, doublet, triplet)

\[ \Delta a_{\mu} \equiv a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = (29.3 \pm 8.4) \rightarrow \begin{cases} 
  m_{\text{NP}} \sim \text{few} \times 100 \text{ GeV} \\
  g_{\text{NP}} \sim 1
\end{cases} \]

\[ \rightarrow \text{Within reach of LHC!} \]

- Identify parameter space that matches $\Delta a_{\mu}$
- Compare with constraints from LHC searches
Two new fields: Allowed parameter space

Freitas, Kell, Lykken, Westhoff ’14
Summary

- **Electroweak precision tests** have played an important role in testing the Standard Model
  - Constraints on $m_t$ and $M_H$ before their discovery

- Today they probe physics beyond the Standard Model at **TeV scale**

- **Electroweak fits** rely on detailed theory calculations

- Intriguing deviation in the **muon anomalous magnetic moment** will be tested in near future
Backup slides